No.

Date

Proposition 3.1. *

Suppose V is a finite-dimensional vector space, $\dim V = n$. Then $\dim \mathcal{L}(V) = n^2$.

Proof.

Let $v_1, \dots, v_n$ be a basis of V. Define $T_{i,j} \in \mathcal{L}(V)$ such that $T_{i,j}(v_i) = v_j$, $T_{i,j}(v_k) = 0$ when $k \neq i$, $1 \leq i, j, k \leq n$. Now we prove $T_{i,j}$'s are linearly independant and $\operatorname{span}(T_{i,j}) = \mathcal{L}(V)$.

Consider the equation $c_{1,1}T_{1,1} + c_{1,2}T_{1,2} + \dots + c_{1,n}T_{1,n} + \dots + c_{n,1}T_{n,1} + \dots + c_{n,n}T_{n,n} = 0$. Applying both sides to v_1 yields $c_{1,1}v_1 + \dots + c_{1,n}v_n = 0$. Since $v_1, \dots, v_n$ is linearly independent, $c_{1,1} = \dots = c_{1,n} = 0$. Apply both sides to $v_2, \dots, v_n$ can, similarly, yield $c_{1,1} = \dots = c_{1,n} = \dots = c_{n,1} = \dots = c_{n,n} = 0$. Thus, $T_{i,j}$'s is linearly independent.

Now, let $T \in \mathcal{L}(V)$ defined as $Tv_k = w_k$, $k \in \{1, \dots, n\}$ and $w_1, \dots, w_n$ is a list of vectors in V (not neccessarily different). Since $v_1, \dots, v_n$ is a basis, we denote each $w_k = b_{k,1}v_1 + b_{k,2}v_2 + \dots + b_{k,n}v_n$. Thus, $T = b_{1,1}T_{1,1} + b_{1,2}T_{1,2} + \dots + b_{1,n}T_{1,n} + \dots + b_{n,1}T_{n,1} + \dots + b_{n,n}T_{n,n} \in \operatorname{Span}(T_{i,j})$. This implies $\operatorname{span}(T_{i,j}) = \mathcal{L}(V)$.

$\square$